

Signatures of confinement in Landau gauge QCD

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Abstract. We summarise an analysis of the infrared regime of Landau gauge QCD by means of a flow equation approach [1]. The infrared behaviour of gluon and ghost propagators is evaluated. The results provide further evidence for the Kugo-Ojima confinement scenario. We also discuss their relation to results obtained with other functional methods as well as the lattice.

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In gauge fixed formulations of QCD the infrared behaviour of ghost and gluon propagators provide signatures of confinement: in Landau gauge QCD the confinement scenarios of Kugo-Ojima [2] and Gribov-Zwanziger, e.g. [3], entail an infrared enhancement for the ghost propagation and an infrared suppression for the gluon propagation. This behaviour was first seen within a functional approach using Dyson-Schwinger equations (DSE) [4], giving access to the full momentum regime. It was later confirmed within lattice studies down to scales about 1 GeV. However, in the deep infrared below 1 GeV, lattice studies encounter problems due to finite size effects. This situation calls for an independent confirmation of the infrared behaviour seen in DS-studies. Ideally such a method would still share enough structure with other functional methods such as DSEs in order to benefit from insights and results obtained from these equations. The above features are precisely given for the flow equation, a particular advantage of which is its flexibility when it comes to approximations. So far this approach has been used in Landau gauge QCD for high and intermediate momenta [5].

Here we present results of a flow equation approach to the infrared regime of Landau gauge QCD [1]. Our analysis is based on an integrated flow equation that reads for the scale-dependent effective action Γ_k ,

$$\Gamma_0 - \Gamma_k = \frac{1}{2} \int_0^k dk \operatorname{Tr} \frac{1}{\Gamma_k^{(2)} + R} \partial_k R. \quad (1)$$

The scale k is an infrared scale below which Γ_k has no propagating degrees of freedom and $\Gamma^{(n)}$ stands for its n th derivative w.r.t. the fields. Consequently $\Gamma = \Gamma_0$ is the full effective action. Flows for Green functions are obtained by taking field derivatives of (1). The resulting equations share many features with DSEs and stochastic quantisation for Green functions. However, in contradistinction to those approaches built on dressed *and* bare quantities, the flow equation (1) links dressed vertices and propagators exclusively. Moreover, the flow equation and its k -integral (1) are manifestly ultraviolet and

infrared finite, no additional renormalisation is required even within truncations. Therefore the flow equation offers an interesting functional method for accessing the infrared regime: its close relation to DSEs makes many of the truncation schemes and results of DSEs accessible to flow equation studies; its qualitative differences and complementary advantages provide additional support for results obtained within both approaches.

We evaluate the integrated flow (1) of ghost and gluon propagators in the deep infrared

$$p^2 \ll \Lambda_{\text{QCD}}^2, \quad (2)$$

where Λ_{QCD} is the dynamical mass scale of QCD. For these momenta the integrated flow (1) tends to zero as the flow reaches a (trivial) fixed point at $k = 0$. This enables us to study the leading infrared behaviour of the propagators by means of a fixed point argument developed in [1]. For momenta in the regime (2) and for $k^2 \ll \Lambda_{\text{QCD}}^2$ we can expand n -point functions at finite k about that at $k = 0$ with

$$\Gamma_k^{(n)} = \Gamma_0^{(n)} (1 + \delta Z_n), \quad (3)$$

where δZ_n only depend on ratios p_i/k with $i = 1, \dots, n-1$. Eq. (3) is valid up to order p_i/Λ_{QCD} . Indeed, (3) can be proven by iterating the integrated flow (1) about $\delta Z_n \equiv 0$. So far we have not relied on any approximation. For the explicit computation we resort to a truncation with dressed vertices with trivial momentum dependence, an approximation which is well in accord with consistency considerations [6] as well as lattice studies [7]. We allow for a general momentum dependence on $x = p^2/k^2$ in the ghost and (transversal) gluon two point functions $\Gamma_{k,C}^{(2)}$ and $\Gamma_{k,A}^{(2)}$

$$\begin{aligned} \Gamma_{k,C}^{(2)}(x) &\simeq z_C p^2 x^{\kappa_C} (1 + \delta Z_C(x)), \\ \Gamma_{k,A}^{(2)}(x) &\simeq z_A p^2 x^{-2\kappa_C} (1 + \delta Z_A(x)), \end{aligned} \quad (4)$$

where we dropped the Lorentz and group structure of the propagators. In (4), $\Gamma_0^{(2)} = z x^\kappa = \hat{z} p^{2\kappa}$ are the leading infrared terms for $k = 0$ with k -independent prefactor \hat{z} . The functions δZ entail the transition between the physical infrared regime, $x \gg 1$, and the cutoff regime, $x \ll 1$. In (4) we have also used that non-renormalisation of the ghost-gluon vertex at vanishing k entails $\kappa_A = -2\kappa_C$ and $\alpha_s = g^2/(4\pi z_A z_C^2)$ [8]. Inserting the propagators (4) in (1) leads to two integral equations for δZ_A and δZ_C of the form

$$\delta Z_{A/C}(x) = F_{A/C}[\delta Z_{A/C}, \kappa_C, \alpha_s]. \quad (5)$$

Explicit expressions for the integrals F are given in [1]. The equations (5) are solved iteratively for δZ_A , δZ_C , κ_C and α_s , leading to

$$\kappa_C = 0.59535 \dots, \quad \alpha_s = 2.9717 \dots. \quad (6)$$

The values in (6) are achieved by also invoking an optimisation procedure developed in [1] for eliminating the regulator-dependence.

The results (6) agree with the analytic results obtained within DSEs [8] and stochastic quantisation [3]. For the present truncation within an optimised cut-off scheme one can

indeed formally link the integrated flow (1) to a set of DSEs with explicit renormalisation, see [1]. We expect a mild R -dependence for the results as we have resorted to a truncation. Indeed, for general cut-off functions R , the results for κ_C mildly vary in the interval

$$\kappa_C \in [0.539, 0.595]. \quad (7)$$

Both bounds have physical interpretations. The upper bound, as we have already discussed, relates to the physical infinite volume result whereas the lower bound can be linked to a finite volume computation. The related regulator is a sharp cut-off that strictly allows no propagation of modes with momenta p^2 smaller than k^2 . This is as close to a finite volume (e.g. in lattice studies) as one can get with local momentum cut-off functions. Interestingly, the lower bound $\kappa_C = 0.539$ compares well to very recent lattice results [9]. A further interesting consequence of our analysis is the evaluation of renormalisation procedures for DSEs stemming from the integrated flow (1): for DSEs in the present truncation and subject to a general consistent renormalisation it is impossible to achieve a masslike behaviour for the gluon propagator, $\kappa_C = 1/2$. More generally $\kappa_C \notin [0, 1/2]$, see [1]. With multiplicative renormalisation this was already shown in [8].

The above analysis allows for many interesting extensions. The most important open question in the present truncation concerns the detailed analysis of the transition between ultraviolet and infrared regime. Unfortunately this question has not been resolved completely in subsequent flow studies [10]. Moreover dynamical quarks are investigated which opens the door towards a description of dynamical chiral symmetry breaking.

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